# MGSC 1205

# Quantitative Methods I

Slides 4 – LP II: Solver, Formulation Application and Excel

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# Minimization LP Problem

Many LP problems involve ***minimizing an objective*** such as ***cost*** instead of maximizing profit function.

**Examples:**

- Restaurant may wish to develop work schedule to meet staffing needs while ***minimizing*** *total number of employees*.
- Manufacturer may seek to distribute its products from several factories to its many regional warehouses in such a way as to ***minimize*** *total shipping costs*.
- Hospital may want to provide its patients with a daily meal plan that meets certain nutritional standards while ***minimizing*** *food purchase costs*.

# Example: Holiday Meal Turkey Ranch

- Buy two brands of feed for good, low-cost diet for turkeys.
- Each feed may contain three nutritional ingredients (protein, vitamin, and iron).

✓ One pound of **Brand A** contains:
  - ✓ 5 ounces of protein,
  - ✓ 4 ounces of vitamin, and
  - ✓ 0.5 ounces of iron.

✓ One pound of **Brand B** contains:
  - ✓ 10 ounces of protein,
  - ✓ 3 ounces of vitamins, and
  - ✓ 0 ounces of iron.

✓ **Minimum monthly requirement per turkey (OZ)**:
Protein: 90, Vitamin: 48, Iron: 1.5

✓ **Brand A** feed costs ranch \$0.02 per pound, while **Brand B** feed costs \$0.03 per pound.

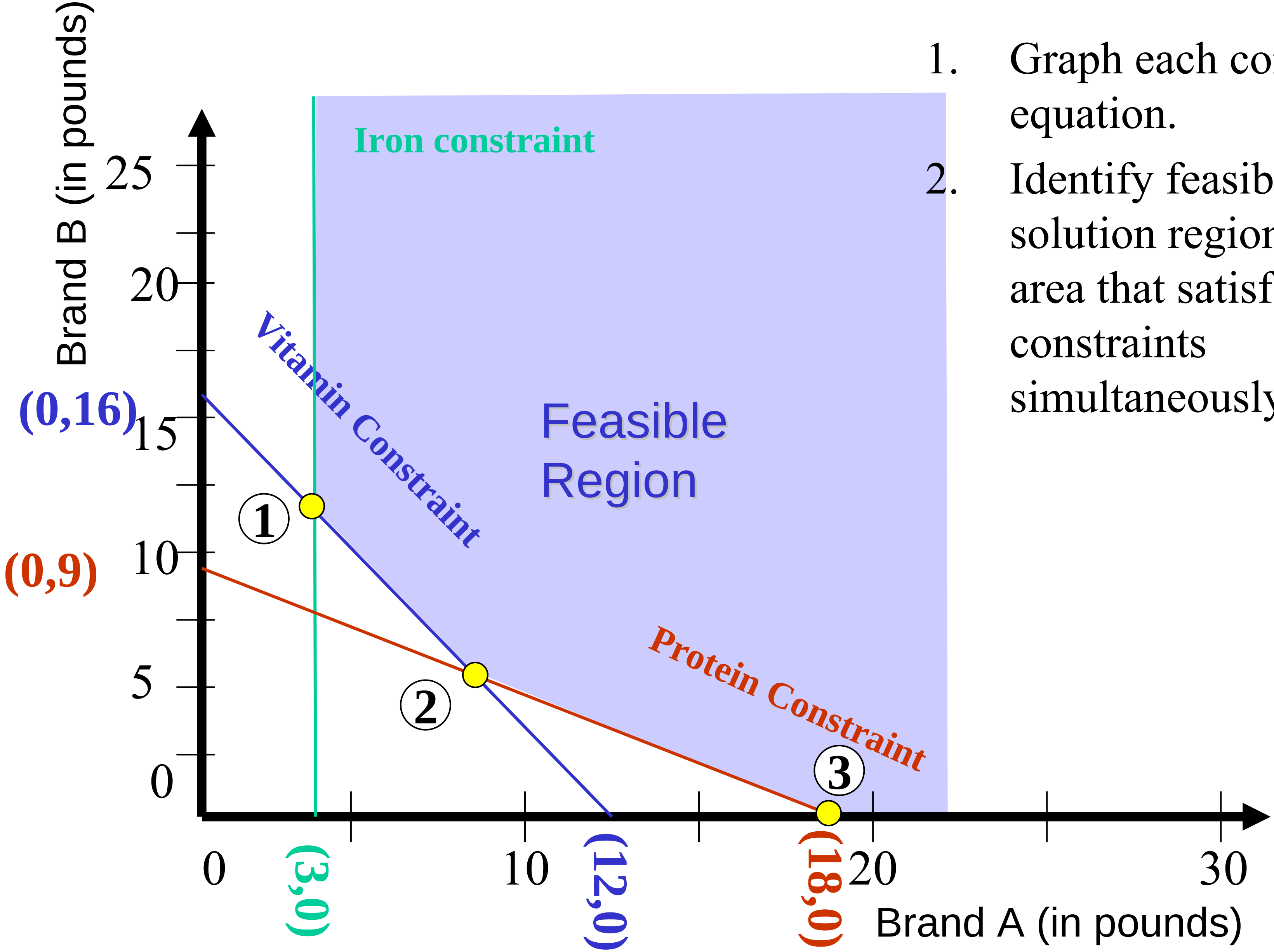
- Ranch owner would like lowest-cost diet that **meets minimum monthly intake requirements** for each nutritional ingredient.

# LP formulation

- Minimize $C = 2\ A + 3\ B$ (in cents)
- or $C = 0.02\ A + 0.03\ B$ (in \$)

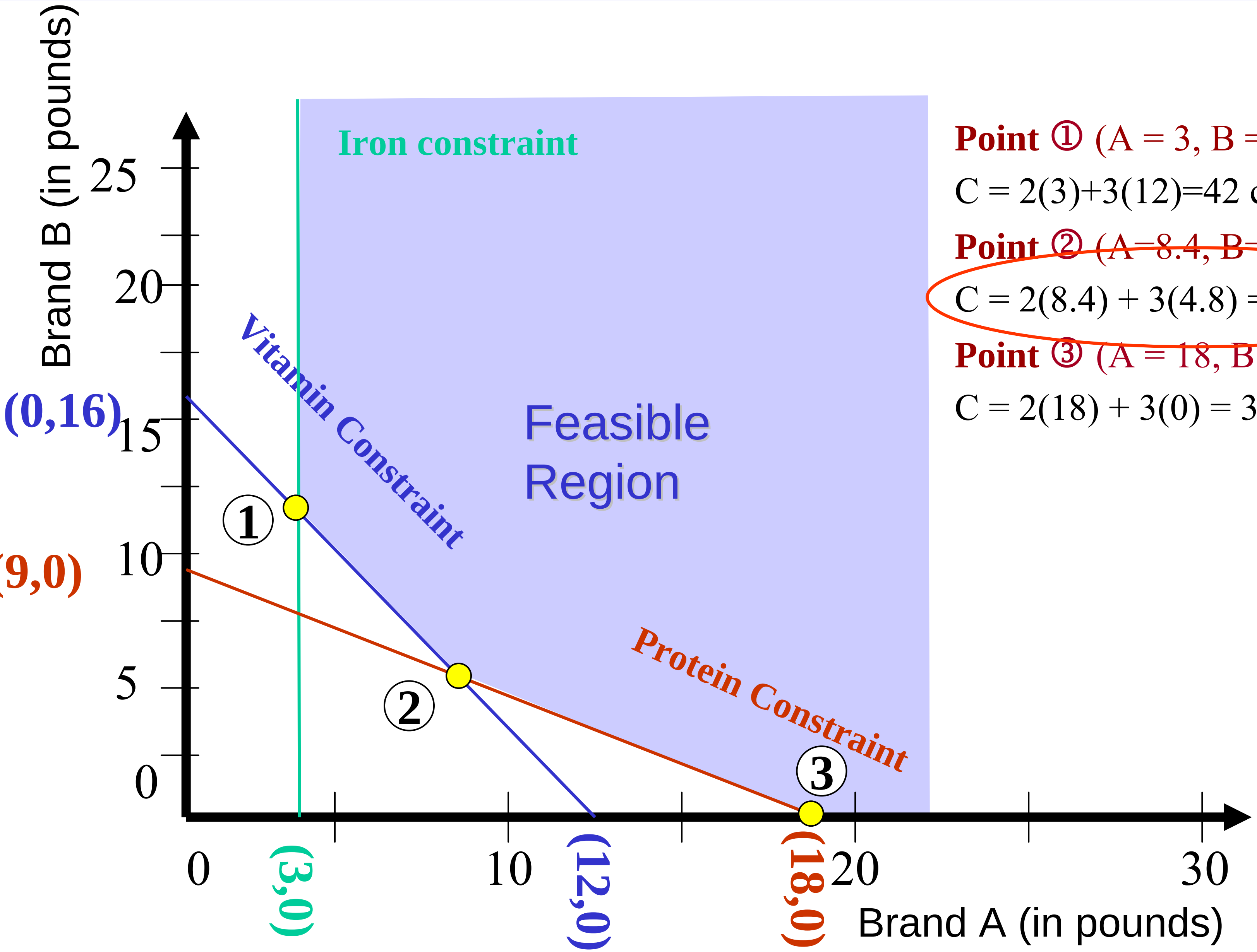
- Subject to:
  - $5\ A + 10\ B \geq 90$ (Protein constraint)
  - $4\ A + 3\ B \geq 48$ (Vitamin constraint)
  - $0.5\ A \geq 1.5$ (Iron constraint)
  - $A \geq 0\ ,\ B \geq 0$ (non-negativity constraints)

# Corner Point Solution Method




1. Graph each constraint equation.
2. Identify feasible solution region, that is, area that satisfies all constraints simultaneously.

# Corner Point Solution Method




**Point ①** (A = 3, B = 12)

C = 2(3)+3(12)=42 cents

**Point ②** (A=8.4, B=4.8)

C = 2(8.4) + 3(4.8) = 31.2 cents

**Point ③** (A = 18, B = 0)

C = 2(18) + 3(0) = 36

# Manufacturing Application

- Anderson Electronics is considering the production of four potential products (VCR, Stereo, TV and DVD)
- Suppose the input for all products can be viewed in terms of just three resources: electronic components, non-electronic components and assembly time.
- The composition of the four products in terms of these inputs is shown in the table below, together with the costs for the resources.
- Each resource is available in limited quantities as shown in the table.
- Due to the market condition, the number of TV produced should be greater or equal to the production of Stereo.
- Formulate the problem as an LP model, which maximizes the total profit.

| | VCR | Stereo | TV | DVD | Supply |
|---|---|---|---|---|---|
| Elct. Components | 3 | 4 | 4 | 3 | 4,700 |
| Non-Elct. Components | 2 | 2 | 4 | 3 | 4,500 |
| Assembly time | 1 | 1 | 3 | 2 | 2,500 |
| Selling price | $70 | $80 | $150 | $110 | |
| Cost | $41 | $48 | $78 | $56 | |
| Profit | $29 | $32 | $72 | $54 | |

- **Maximize**
- $P = 29\,V + 32\,S + 72\,T + 54\,D$
- Or : $R = 70\,V + 80\,S + 150\,T + 110\,D$

  $C = 41\,V + 48\,S + 78\,T + 56\,D$

  $P = (70-41)\,V + (80-48)\,S + (150-78)\,T + (110-56)\,D$
- **Such that:**

➢ $3\,V + 4\,S + 4\,T + 3\,D \leq 4700$ (Elct. Comp. constraint)

➢ $2\,V + 2\,S + 4\,T + 3\,D \leq 4500$ (Non-Elct. Comp. constraint)

➢ $V + S + 3\,T + 2\,D \leq 2500$ (Ass. time constraint)

➢ $S \leq T$ or $S - T \leq 0$ (Market constraint)

➢ $V, S, T, D \geq 0$ (Non-negativity constraint)

## Formulating in Excel

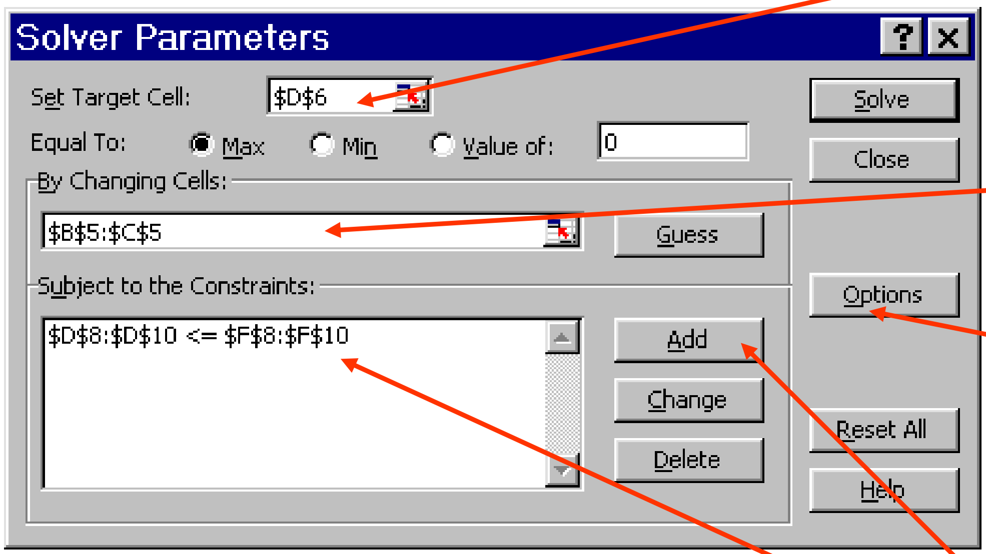
- Write the LP out on paper, with the objective function and all constraints.
- Decide on cells to represent variables.
- Enter parameters of each variable in each constraint in a block of cells.

| **Flair Furniture** | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| | ***T*** | ***C*** | | | |
| | Tables | Chairs | | | |
| Number of Units | 30.0 | 40.0 | | | |
| Profit | $7.00 | $5.00 | $410.00 | **<- Objective** | |
| **Constraints:** | | | | | |
| Carpentry Hours | 4 | 3 | 240.0 | <= | 240 |
| Painting Hours | 2 | 1 | 100.0 | <= | 100 |
| Chairs Limit | | 1 | 40.0 | <= | 60 |
| | | | **LHS** | **Sign** | **RHS** |

# Solving LP Problems Using Excel's Solver

Objective function, referred to as *target cell* by solver

Solver refers to decision variables as *changing cells*.




Options window: One must check boxes titled

- Assume ***Linear Model***
- Assume ***Non-Negative***

Use "Add" constraints to enter relevant cell references for *LHS* and *RHS*.

# Applications

- Marketing Application: Media Selection (p. 84)
- Transportation Problem (Module 3)
- Employee Scheduling Application: Labor Planning (p. 91)
- Financial Application: Portfolio Selection (p. 94)

# (1) Marketing Application: Media Selection (p. 84)

- Kitchener Electronics has budgeted up to \$8,000 per week for local advertising.
- The money is to be allocated among four promotional media:
- TV spots , newspaper ads, and two types of radio advertisements.
- **Kitchener's goal:** is to reach the largest possible audience through various media.
- The following table represents the number of potential customers reached by making use an advertisement in each of the four media.
- It also provides the cost per advertisement placed and the maximum number of ads that can be purchased per week.

| Medium | Audience Reached Per Ad | Cost Per Ads | Maximum Ads Per Week |
|---|---|---|---|
| TV spots | 5000 | \$800 | 12 |
| Newspaper ads | 8500 | \$925 | 5 |
| Prime-time radiospots | 2400 | \$290 | 25 |
| Afternoon radio spots | 2800 | \$380 | 20 |

✓Contract arrangements require at least 5 radio spots be placed per week.

✓Management insists no more than \$1,800 be spent on radio Ads per week.

# Mathematical formulation

- Let
- ➢ T = # 1-minute TV spot each week
- ➢ N = # full-page daily newspaper ads taken each week
- ➢ P = # 30-seconed prime-time radio spots taken each week
- ➢ A = # 1-minute afternoon radio spots taken each week

❖ **Objective**

✓ Maximize audience coverage =

$$5000T + 8500N + 2400P + 2800A$$

✓ **Subject to**

$T \leq 12$ (max TV spots/week)

$N \leq 5$ (max newspaper ads/week)

$P \leq 25$ (max 30-seconed radio spots /week)

$A \leq 20$ (max 1-minute radio spots /week)

$800T+925N+290P+380A \leq \$8000$ (weekly advertising budget)

$P+A \geq 5$ (min radio spots contracted)

$290P+380A \leq \$1800$ (max dollars spent on radio)

$T, N, P, A \geq 0$ (non-negativity conditions)

# (2) Transportation Problem

- Century Plastics operates two injection molding plants in Moncton and Winnipeg. Approximately 50% of their production is custom manufacturing for automakers in Canada and the US. This output takes priority and is shipped directly to the customer factories.
- The remaining capacity is used to make after-market parts that are sold through auto supply firms such as UAP and Canadian Tire. These products are shipped through distribution centres in Montreal, Vancouver and Cleveland.
- Estimated available capacity in the next month is 10,000 tons in Moncton and 20,000 tons in Winnipeg.
- The total requirements in Montreal, Vancouver and Cleveland are 6,000, 8,000 and 16,000 respectively.
- The shipping costs per ton are summarized in the table below:

| | **Montreal** | **Vancouver** | **Cleveland** |
|---|---|---|---|
| **Moncton** | **60** | **120** | **80** |
| **Winnipeg** | **100** | **60** | **90** |

❖ How can Century meet the demands of its distribution centres in the least cost manner?

# Formulation - Solution

- The **decision variables** are how much to ship from each plant to each warehouse.
- Let
- $M_M$ = the number of tons made in Moncton and shipped to Montreal
- $M_V$ = the number of tons made in Moncton and shipped to Vancouver
- $M_C$ = the number of tons made in Moncton and shipped to Cleveland
- $W_M$ = the number made in Winnipeg and shipped to Montreal
- $W_V$ = the number made in Winnipeg and shipped to Vancouver
- $W_C$ = the number made in Winnipeg and shipped to Cleveland
- The **objective** is to minimize costs:

$$\textbf{Minimize } C = 60M_M + 120M_V + 80M_C + 100W_M + 60W_V + 90W_C$$

- **The constraints** are that
  - we cannot ship more than we can make, and
  - we have to ship at least as much as we need.

| Supply | Demand | Non-Negativity |
|---|---|---|
| $M_M + M_V + M_C \leq 10{,}000$ | $M_M + W_M \geq 6{,}000$ | $M_M, M_V, M_C, W_M, W_V,$ |
| $W_M + W_V + W_C \leq 20{,}000$ | $M_V + W_V \geq 8{,}000$ | $W_C \geq 0$ |
| | $M_C + W_C \geq 14{,}000$ | |

# Solution using Excel

Text Box 5 ▼ fx

| | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Century Plastics Transportation** | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | MM | MV | MC | WM | WV | WC | | | | |
| 4 | | Monc-Mtl | Monc-Var | Monc-Cle | Winn-Mtl | Winn-Van | Winn-Clev | | | | |
| 5 | # of units shipped | 6000 | 0 | 4000 | 0 | 8000 | 10000 | Total | | | |
| 6 | Cost | 60 | 120 | 80 | 100 | 60 | 90 | 2060000 | | | |
| 7 | Constraints | | | | | | | | | | |
| 8 | Moncton Supply | 1 | 1 | 1 | | | | 10000 | <= | 10000 | |
| 9 | Winnipeg Supply | | | | 1 | 1 | 1 | 18000 | <= | 20000 | |
| 10 | Montreal Demand | 1 | | | 1 | | | 6000 | >= | 6000 | |
| 11 | Vancouver Demand | | 1 | | | 1 | | 8000 | >= | 8000 | |
| 12 | Cleveland Demand | | | 1 | | | 1 | 14000 | >= | 14000 | |
| 13 | | | | | | | | LHS | sign | RHS | |
| 14 | | | | | | | | | | | |
| 15 | | | | | | | | | | | |
| 16 | | | | | | | | | | | |
| 17 | | | | | | | | | | | |
| 18 | | | | | | | | | | | |
| 19 | | | | | | | | | | | |
| 20 | | | | | | | | | | | |
| 21 | | | | | | | | | | | |

The values of the variables are in cells B5:G5. These are the cells that are **changed** in Solver

Objective is to Minimize Total Cost
H6 is the **Target Cell**

Column H contains the totals for each row. Totals are calculated using =SUMPRODUCT(B?:G?,$B$5:$H:$5) where ? Is replaced with the number for that row. These cells represent the left hand side (LHS) of the constraints in Solver (**cell reference**).

In Solver, the right hand side RHS of the constraint is called **constraint**.

# Answer report

Target Cell (Min)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $H$6 | Cost Total | 0 | 2060000 |

Adjustable Cells

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $B$5 | # of units shipped Monc-Mtl | 0 | 6000 |
| $C$5 | # of units shipped Monc-Van | 0 | 0 |
| $D$5 | # of units shipped Monc-Clev | 0 | 4000 |
| $E$5 | # of units shipped Winn-Mtl | 0 | 0 |
| $F$5 | # of units shipped Winn-Van | 0 | 8000 |
| $G$5 | # of units shipped Winn-Clev | 0 | 10000 |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $H$8 | Moncton Supply Total | 10000 | $H$8<=$J$8 | Binding | 0 |
| $H$9 | Winnipeg Supply Total | 18000 | $H$9<=$J$9 | Not Binding | 2000 |
| $H$10 | Montreal Demand Total | 6000 | $H$10>=$J$10 | Binding | 0 |
| $H$11 | Vancouver Demand Total | 8000 | $H$11>=$J$11 | Binding | 0 |
| $H$12 | Cleveland Demand Total | 14000 | $H$12>=$J$12 | Binding | 0 |

# (3) Employee Scheduling Application: Labor Planning (91)

Atlantic Bank of Canada now employs 12 full-time tellers.

Part-time employees (four hours per day) are available.

- ✓ Part-timers earn \$4 per hour (or \$16 per day) on average.
- ✓ Full-timers earn \$50 per day in salary and benefits, on average.
- Full-timers work from 9am to 5pm
  - Allowed 1 hour for lunch.
  - Half of full-timers eat at 11 A.M. and other half at noon.
  - Full-timers thus provide 35 hours per week of productive labor time.
- Part-times can start anytime between 9am and 1pm; but hour limited to a maximum of 50% of day's total requirement.

- Objective: Minimize total daily personnel cost

$F$ = full-time tellers

$P_1$ = part-timers starting at 9 A.M. (leaving at 1 P.M.)

$P_2$ = part-timers starting at 10 A.M. (leaving at 2 P.M.)

$P_3$ = part-timers starting at 11 A.M. (leaving at 3 P.M.)

$P_4$ = part-timers starting at noon (leaving at 4 P.M.)

$P_5$ = part-timers starting at 1 P.M. (leaving at 5 P.M.)

Required labor hours:

| Time Period | # of Tellers Required |
|---|---|
| 9am - 10am | 10 |
| 10am -11am | 12 |
| 11am - Noon | 14 |
| Noon - 1pm | 16 |
| 1pm - 2pm | 18 |
| 2pm - 3pm | 17 |
| 3pm - 4pm | 15 |
| 4pm - 5pm | 10 |

# Available Labor Hours

| 9 | 10 | 11 | Noon | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| F | F | F/2 | F/2 | F | F | F | F | |
| $P_1$ | $P_1$ | $P_1$ | $P_1$ | | | | | |
| | $P_2$ | $P_2$ | $P_2$ | $P_2$ | | | | |
| | | $P_3$ | $P_3$ | $P_3$ | $P_3$ | | | |
| | | | $P_4$ | $P_4$ | $P_4$ | $P_4$ | | |
| | | | | $P_5$ | $P_5$ | $P_5$ | $P_5$ | |

| Time Period | # of Tellers Required |
|---|---|
| 9am - 10am | 10 |
| 10am -11am | 12 |
| 11am - Noon | 14 |
| Noon - 1pm | 16 |
| 1pm - 2pm | 18 |
| 2pm - 3pm | 17 |
| 3pm - 4pm | 15 |
| 4pm - 5pm | 10 |

# Mathematical formulation

- **Objective**
- ✓ Minimize total daily personnel cost =

$$\$50\,F + \$16\,(P_1+P_2+P_3+P_4+P_5)$$

- ✓ **Constraints (Subject to)**

**For each hour, the available labour must equal to the required labour hours.**

$F+P_1 \geq 10$ (9am-10am needs)

$F+P_1+P_2 \geq 12$ (10am-11am needs)

$\frac{1}{2}F+P_1+P_2+P_3 \geq 14$ (11am-noon needs)

$\frac{1}{2}F+P_1+P_2+P_3+P_4 \geq 16$ (noon-1pm needs)

$F+P_2+P_3+P_4+P_5 \geq 18$ (1pm-2pm needs)

$F+P_3+P_4+P_5 \geq 17$ (2pm-3pm needs)

$F+P_4+P_5 \geq 15$ (3pm-4pm needs)

$F+P_5 \geq 10$ (4pm-5pm needs)

Only 12 full-time tellers are available, so

$$F \leq 12$$

Part-time hours cannot exceed 50% of total hours required each day, so

$$4(P_1+P_2+P_3+P_4+P_5) \leq 0.50(10+12+14+18+17+15+10) = 56$$

$$F,\ P_1,P_2,P_3,P_4,P_5 \geq 0 \text{ (non-negativity conditions)}$$

# (4) Financial Application: Portfolio Selection (p. 94)

- International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans.
- ICT has $5 million available for immediate investment  and wishes to do two things:
  - maximize interest earned on investments made over next six months
  - satisfy diversification requirements as set by  board of directors.
- Investment Possibilities:

| Investment | Interest earned% | Maximum investment ($ Millions) |
|---|---|---|
| Trade credit | 7% | 1.0 |
| Corporate bonds | 11% | 2.5 |
| Gold stocks | 19% | 1.5 |
| Construction loans | 15% | 1.8 |

✓Board specifies at least 55% of funds invested must be in gold stocks and construction loans.

✓No less than 15% be invested in trade credit.

# Mathematical formulation

- Let
- T = dollars invested in trade credit
- B = dollars invested in corporate bonds
- G = dollars invested in gold stocks
- C = dollars invested in construction loans
- **Objective**
- ✓ Maximize dollars of interest earned =

$$0.07\,T + 0.11\,B + 0.19\,G + 0.15\,C$$

- **Subject to**

$T \leq 1{,}000{,}000$ (max investment in T)

$B \leq 2{,}500{,}000$ (max investment in B)

$G \leq 1{,}500{,}000$ (max investment in G)

$C \leq 1{,}800{,}000$ (max investment in C)

$G + C \geq 0.55\,(T+B+G+C)$ (Board condition 1)

$T \geq 0.15\,(T+B+G+C)$ (Board condition 2)

$T+B+G+C \leq 5{,}000{,}000$ (max dollars invested)

$T, B, G, C \geq 0$ (non-negativity conditions)